HCN-003-001515

Seat No. ________

B. Sc. (Sem. V) (CBCS) Examination

October - 2017

Mathematics : Paper - 503 (A)

(Discrete Mathematics & Complex Analysis - I)

Faculty Code : 003
Subject Code : 001515

Time : $2\frac{1}{2}$ Hours] [Total Marks : **70**

Instructions : (1) All the questions are compulsory.
(2) Numbers written to the right indicate full marks of the question.

1 Attempt all the questions : **20**

(1) State Idempotent law.

(2) What is the value of the sum of all minterms in n-variable?

(3) What is the value of $m_i * m_j$ if m_i and m_j are distinct minterms in n variables.

(4) State modular inequality.

(5) If a is an atom of a Boolean algebra $(B, *, \oplus, ', 0, 1)$ then $\forall x \in B,$ what are the possible values of $a * x$?

(6) State Isotonicity property.

(7) How many squares are there in a Karnaugh map of an expression containing three variables?

(8) State absorption law.

(9) Write maximal elements for the POSET $(\{2,3,4,6\}, D)$.

(10) What is the value of $a * 1$ in a bounded lattice $(L, *, \oplus, 0, 1)$?

(11) Write formula to find the length of smooth arc.

(12) State Liouville's theorem.

(13) What is the value of $\int_c \frac{z^2}{z-1} dz; c : |Z| = 2.$

(14) Write Cauchy Riemann condition for complex function $f(z) = u + iv$ to be analytic.

(15) State fundamental theorem for algebra.

(16) What is the value of $\lim_{z \to \infty} \frac{2z+3}{z+i}$.

(17) Write Laplace equation.

(18) State Cauchy's inequality.

(19) Write Cauchy Riemann condition in polar form for complex function $f(z)$ to be analytic.

(20) Write the formula to find $f'(z_0)$ for a complex function $f(z) = u + iv$.

2 (A) Attempt any **three** : **6**

(1) If a and b are distinct atoms of a Boolean algebra $(B, *, \oplus, ', 0, 1)$ then prove that $a * b = 0$.

(2) For a non-empty set A if $X, Y \in P(A)$, $X * Y = X \cap Y$, $X \oplus Y = X \cup Y$ then prove that $(P(A), *, \oplus)$ is a Lattice.

(3) Define greatest element and maximal element in a POSET.

(4) Obtain Minimal sum of product of the Boolean expression by $\alpha(x, y) = xy + xy'$ by Karnaugh map.

(5) If $(B, *, \oplus, ', 0, 1)$ is a finite Boolean algebra and x is a nonzero element of B then prove that there exists an atom a in B such that $a \le x$.

(6) Draw the Hasse diagram of (S_{60}, D).

(B) Attempt any **three** : **9**

(1) State and prove distributive inequalities.

(2) Prove that every chain is a distributive lattice.

(3) In usual notation prove that 0 and 1 are the unique complements of each other.

(4) If $(B,*,\oplus,',0,1)$ is a Boolean algebra then prove that for any $x_1, x_2 \in B$ $A(x_1 \oplus x_2) = A(x_1) \cup A(x_2)$.

(5) State and prove De Morgan's law for Boolean algebra.

(6) Express the Boolean expression $(x_2 * x_3)$ as the product of its maxterms.

(C) Attempt any **Two** : **10**

(1) State and prove Stone's representation theorem of Boolean algebra.

(2) Prove that in a complemented distributive lattice, the following are equivalent.

(i) $a \le b$ (ii) $a \wedge b' = 0$

(iii) $a' \vee b = 1$ (iv) $b' \le a'$

(3) If $(L,*,\oplus)$ is a lattice then prove that for any $a,b \in l$, $\text{glb}\{a,b\} = a * b$ and $\text{lub}\{a,b\} = a \oplus b$, with respect to partial ordering R on L.

(4) State and prove Unique representation theorem of Boolean algebra.

(5) Define direct product of two lattices and prove that it is a lattice.

3 (A) Attempt any **three** : **6**

(1) Prove that $f'(z) = f(z)$, *where* $f(z) = e^z$.

(2) Define : (i) Analytic Function (ii) Entire Function

(3) Evaluate : $\int_0^{2+i} z^2 dz$.

(4) Prove that $u = \sinh x \sin y$ is harmonic.

(5) Evaluate : $\int_C \frac{dz}{z^2+4}; C : |z-i| = 2.$

(6) Define : (i) Jordan arc (ii) Contour.

(B) Attempt any **three** : **9**

(1) Show that the function $\left(\frac{1}{z}\right)$ is analytic but not entire.

(2) If u and v are conjugate harmonic functions then prove that the family of curves obtained by $u = c_1$ and $v = c_2$ are orthogonal.

(3) Show that the Cauchy Riemann conditions are sufficient conditions for a complex function $f(z)$ to be analytic.

(4) Evaluate : $\int_C \frac{dz}{(z-1)(z-2)}; C : |z| = 3.$

(5) In usual notation prove that $\left|\int_a^b f(z)\,dz\right| \le \int_a^b |f(z)|\,dz.$

(6) State and prove Cauchy's fundamental theorem.

(C) Attempt any **Two** : **10**

(1) Obtain Laplace equation in polar form.

(2) Find an analytic function $f(z) = u + iv$ such that $u - v = x + y.$

(3) State and prove Morera's theorem.

(4) Show that $u = 2x(1-y)$ is a harmonic function and find its conjugate.

(5) State and prove Cauchy's integral formula.